Optimisation of tidal turbine arrays

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Tidal array modelling

The regularity of the tides makes them very attractive for renewable power production. The UK is particularly suited for tidal generation, with a technical resource supply of about 116 TWh/year [1], thanks to its bathymetry and coastline features which accelerate tidal currents in many locations.
However, the first full-sized arrays will only be developed if they can be shown to be viable from an economic, engineering and environmental perspective. Advanced numerical tools are needed to both predict and maximise power yield, in order to prove viability of new sites and aid array design in this fledgling industry.
Substantial progress on the development of such tools has been made in recent years. In this project we aim to further this work in a number of ways:

- Comparison of different turbulence models, which parameterise subgrid-scale processes and may enable an improvement in accuracy without increasing computational expense.
- Refinement of economic models used for tidal arrays and investigation of the impact that optimising with respect to different economic indicators has on the array design.
- Extension of the tools to include, optimisation of the operation of the turbines once installed. For example, dynamic control of the blade pitch of individual turbines may allow the array to mitigate the effects of device down-time and reduce structural loads and need for maintenance.

Figure: The installation of the world's first array of tidal turbines. Meygen have installed 3 in Pentland Firth, whereas full-sized arrays may comprise of hundreds of turbines. [2]

Importance of wake modelling

There is a cubic relationship between power and flow speed, $P = \frac{1}{2}\rho A u^3$, so it is important to locate the turbines in optimal areas. Turbines obstruct the flow and cause turbulent wakes, characterised by lots of eddies, to form behind them. The acceleration adjacent to the swept area and deceleration within it along with the flow seperation leads to turbulent mixing layers forming [3]. The eddies in the mixing layer transfer momentum from the fast flow outside of the swept area's downstream cross-section to the slower flow within it, causing the velocity of the cross section to even out far enough downstream. This is a complex multi-scale problem and it is necessary to model the flow speeds accurately in order to avoid placing turbines in the slow and turbulent wakes of the upstream devices.

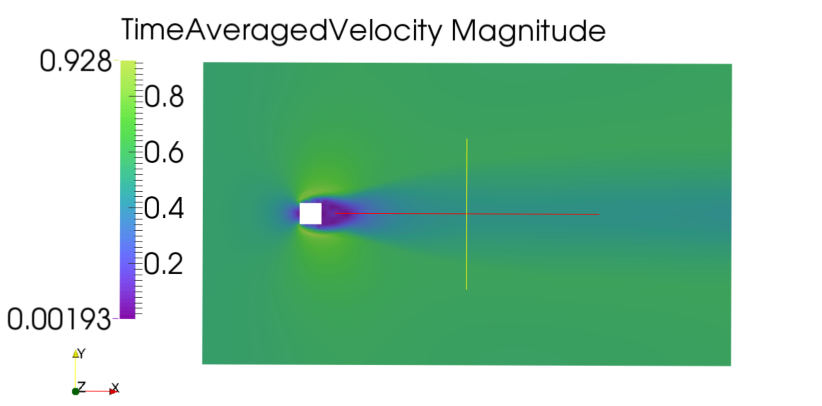


Figure: Normalised flow past a square simulating wake of a tidal turbine, in Thetis.

Different types of turbulence model

Numerical solutions are not able to explicitly capture the effects of small scale turbulence on their own unless solved on such a fine mesh that all turbulent length scales are resolved through direct numerical simulation (**DNS**). This is often far too computationally expensive to be feasible. Instead turbulence models are used to compensate for the effect of subgrid scale features.
LES: In large eddy simulation the unresolved turbulent eddies are compensated for by adding an extra term called the subgrid scale viscosity. LES therefore updates its viscosity after each time step so it is the sum of the molecular viscosity and the subgrid scale viscosity calculated at that time step.

$$\nu = \nu^{\text{mol}} + \nu^{\text{SGS}}, \tag{1}$$

$$\nu^{\text{SGS}} = \rho (C_s \triangle)^2 |\bar{S}|, \tag{2}$$

where $\nu^{mol} = 10^{-6} m^2 s^{-1}$ is the kinematic molecular viscosity for water, S is the rate of deformation tensor found from $S = \frac{1}{2}(\nabla \underline{u} + \nabla \underline{u}^T)$, $\triangle$ is the filter width, taken as a multiple of the mesh size, and C_s is the Smagorinsky constant.
HLES: the viscosity consists of an extra term, to compensate for unresolved 3D effects on the quasi-2D turbulence. HLES consists of up to three viscosity terms, the Elder term being optional.

$$\nu = \nu^{\text{mol}} + \nu^{\text{SGS}} + \nu^{\text{Elder}}, \tag{3}$$

$$\nu^{\text{SGS}} = \frac{1}{k_s^2}\left(\sqrt{(\gamma \sigma_T S^*)^2 + B^2} - B\right) \quad \text{and} \quad B = \frac{3}{4} C_f \frac{|\underline{u}|}{h} \tag{4}$$

The additional viscosity is defined by the Elder formula,

$$\nu^{\text{Elder}} = \frac{1}{6} \kappa u_* (h + \Delta\eta), \tag{5}$$

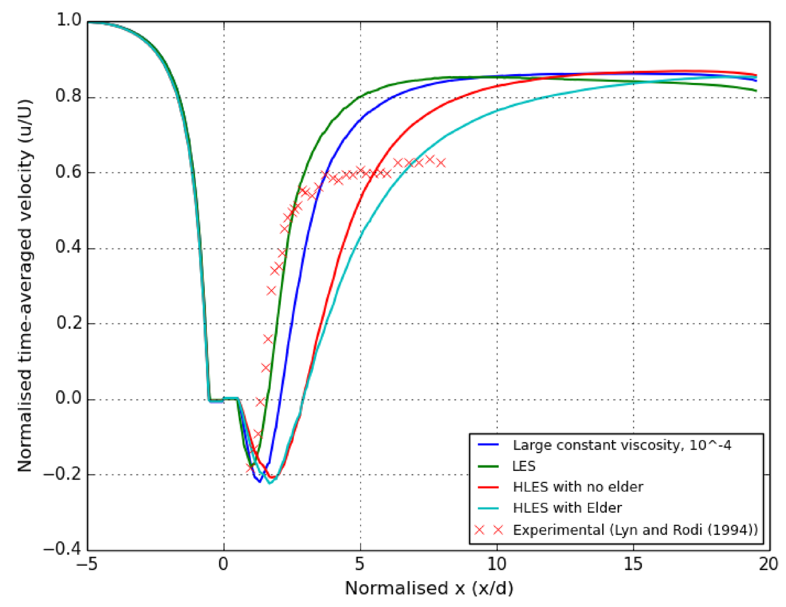
where $u_* = \sqrt{C_f} \cdot |\underline{u}|$ is the bed-friction velocity that depends on the bottom drag coefficient, C_f, and κ is the Von Karman constant [4].

References

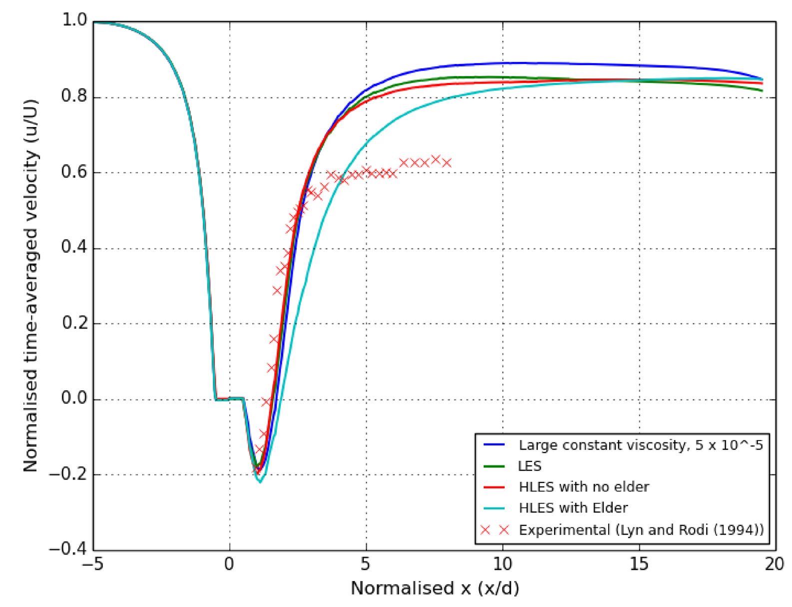
[1] "UK tidal current resource in the crown estate's key resource areas", February 2013.
[2] S. Arnaud, "First fully operational tidal array deployed in Shetland", August 2016.
[3] S. Mungar, "Hydrodynamics of horizontal-axis tidal current turbines", January 2014.
[4] H. Talstra, "Large-scale turbulence structures in shallow separating flows", May 2011.

Comparison of 2D turbulence models

Simulations of flow past a square, to represent a tidal turbine, show that after optimising the parameters of each of the turbulence models all of the results are accurate in the near wake, however they overestimate the wake regeneration in the far wake. Further adaptations to the turbulence models and numerical methods used need to be implemented to mitigate this. The results then must be compared to 3D models do identify whether the tidal array optimisation problem can be approximated in 2D to save computational expense, or if the losses in accuracy are too severe.



(a) Comparison of turbulence models with parameters set from literature



(b) Comparison of turbulence models with parameters optimised

Economic Indicators

Rather than just optimising power, the tools for tidal arrays can be adapted to include economic indicators in the functional. This can make the work more relevant for industry by providing optimised solutions that appropriately balance maximising energy generation and minimising capital costs relating to number of devices and cable length and operational costs, which may be more expensive in deeper waters and further offshore.
Many metrics used in the financing of renewable energy rely on the principle of discounting.
Discounted cash flow analysis (DCF) is a way of quantifying the idea that money today is worth more than money tomorrow, given that current funds have the ability to earn interest and become worth more in the future, to enable the comparison of cost in different time periods.
Net Present Value (NPV) is the sum of all incoming and outgoing cashflows, adjusted according to the time value of money.

$$NPV = \sum_{t=0}^{N} \frac{E_t \cdot T_t + C_t}{(1+r)^t} \tag{6}$$

where r is the discount rate, C_t denotes costs at year t, E_t denotes the energy generation in year t and T_t denotes the agreed energy tariff for that year. C_0 is therefore the CAPEX and C_t where $t \in [1, n]$ is the OPEX for each year of the tidal development.
Levelised Cost of Energy (LCOE) is a proxy for the average price of energy that an array must receive to break even over its lifetime. It is calculated by finding the NPV of the unit-cost of electricity over the lifetime of the array.

$$LCOE = \frac{discounted\ cost}{discounted\ energy} = \frac{\sum_{t=0}^{N} \frac{C_t}{(1+r)^t}}{\sum_{t=0}^{N} \frac{E_t}{(1+r)^t}} \tag{7}$$

Internal Rate of Return (IRR) is the discount rate, r, that makes the NPV of a project equal to zero. It is used as a measure of profitability of potential investments. This results in an N^{th} degree polynomial where N is the number of years in the project lifetime. It cannot be solved analytically so is frequently found through the Secant method.

$$r_{n+1} = r_n - NPV_n \cdot \left(\frac{r_n - r_{n-1}}{NPV_n - NPV_{n-1}}\right) \tag{8}$$

This indicators are being implemented into Thetis. The impact of optimising with respect to different indicators due to different stakeholder incentives is being investigated.