# Particle Filtering for Sandstorm Tracking

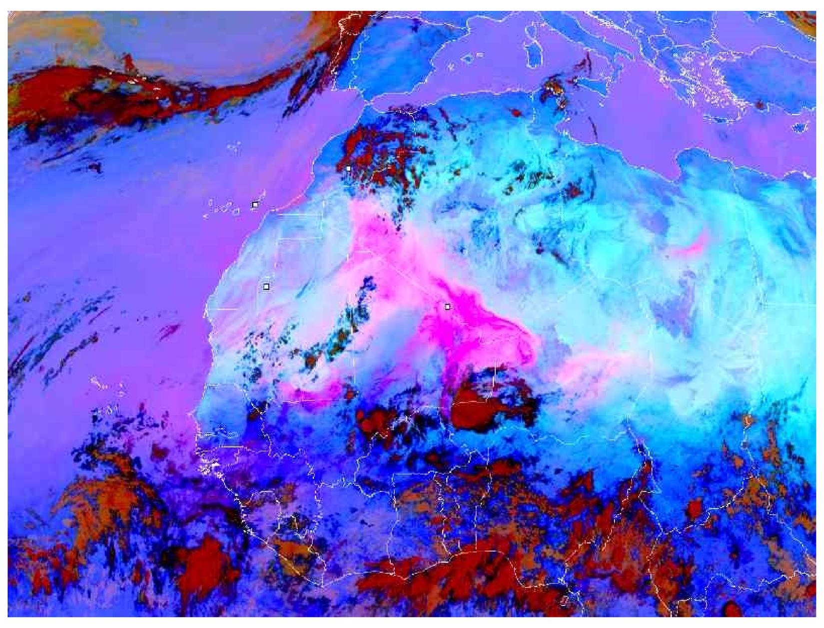
Francesc Pons Llopis


## Abstract

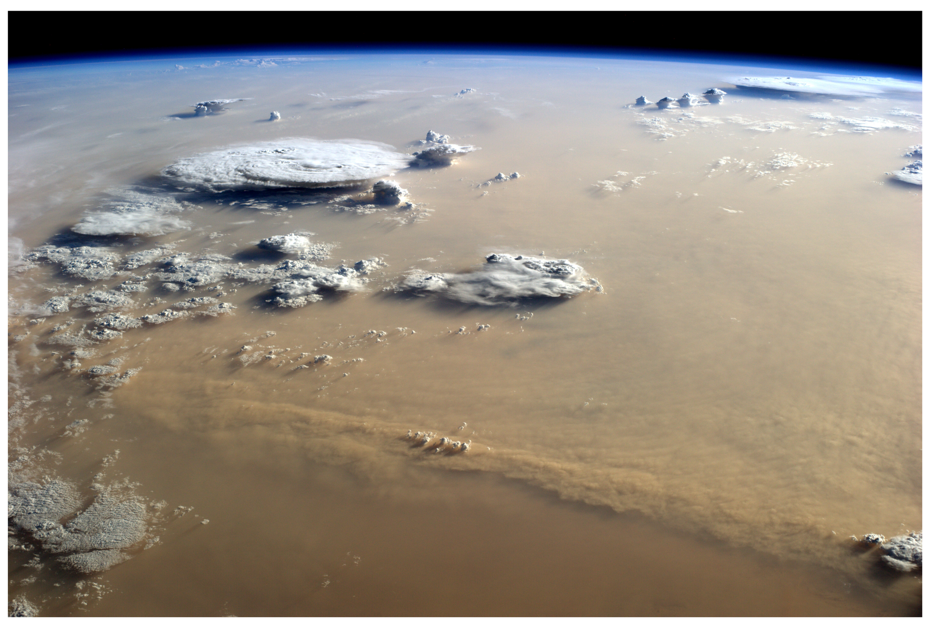
Data Assimilation combines the model of the dynamics of a system together with sequentially arriving observations in order to provide estimates of the hidden states of a system. In addition, it is of great importance to quantify the uncertainty of these estimates. Particle filters are a generic method to address this problem in a principled way. So far applying this methodology to high dimensional problems in geophysical sciences has been challenging. In this project, we aim to develop particle filters that could be applied in such a scenario and in particular for tracking sand storms in the Sahara dessert.


## Why tracking sandstorms?

- Saharan dust storms have a strong impact on the Earth's energy budget.
- Infra-red imagery can be used to highlight the presence of sand storms and track their movement through space and time.
- Methods used so far for tracking are usually ad-hoc and subjective, relying on expert knowleldge and simplistic modelling assumptions.
- We aim to develop an approach for identifying and tracking dust storms from observations by a Spinning Enhanced Visible and InfraRed Imager (SEVIRI).



Sahara Desert image with infraRed Imager



Source: NASA

## Bayesian Inference and Stochastic Filtering

- Let $V_t$ denote the **unknown** space-time varying 2D velocity wind field on the Sahara desert.
  - The signal $V_t$ is modelled by the stochastic Navier Stokes equation.
- Let $Y_t$ also denote the non-linear and noisy observations of $V_t$. In general, these can be of different types:
  - Eulerian: wind field measured in fixed points of the surface.
  - Lagrangian: measurements from moving sensors according to the wind field.
  - Other nonlinear observation such as the satellite imaging.
- **Stochastic Filtering**: We want to compute the following posterior filtering distribution as given by Bayes rule

$$\mathcal{L}aw(V_{0:t}|Y_{0:t}) \propto \mathcal{L}aw(Y_{0:t}|V_{0:t})\mathcal{L}aw(V_{0:t}).$$

- This can be implemented recursively using a sequence of prediction and correction steps, see [1] for more information.

## Modelling the dynamics of the signal

### Model of the signal

- Dust-storm evolution is controlled by the wind.
- Evolution of atmosphere is modelled by the incompressible 2D Navier-Stokes (NS) equation.

$$\partial_t v - \nu\Delta v + (v \cdot \nabla)\, v = f - \nabla p,\ v(x,0) = v_0(x),$$
$$\nabla \cdot x = 0,\ \int v_j(x,\cdot)dx = 0$$

where $f$ is the forcing, $v$ the vector field, $p$ the pressure and $\nu$ the viscosity coefficient.

- We will use stochastic forcing to account for
  - model uncertainty
  - unresolved scales
- We can project NS solution to a suitable basis, i.e. use weak form solution of the type:

$$dV_t + \nu A V_t + F(V_t) = dW_t$$

where $W_t$ is space-time white noise, $A$ is a linear operator (projected Laplacian), and $F$ advection non-linearity.

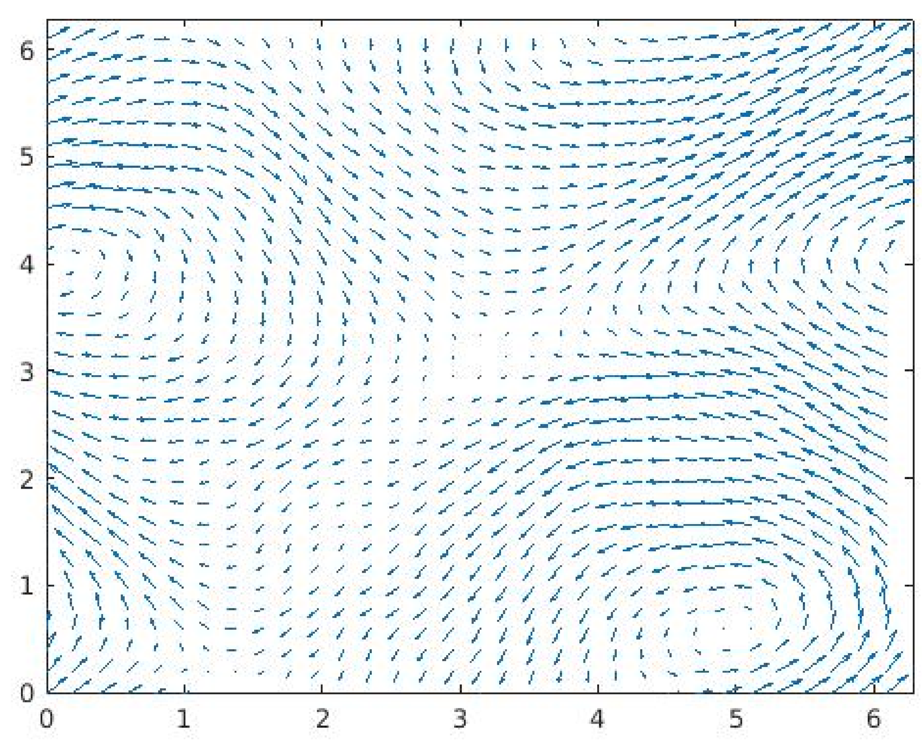
Simulation from this signal requires appropriate numerical scheme as in [2].

### Numerical Solution of Stochastic Navier-Stokes

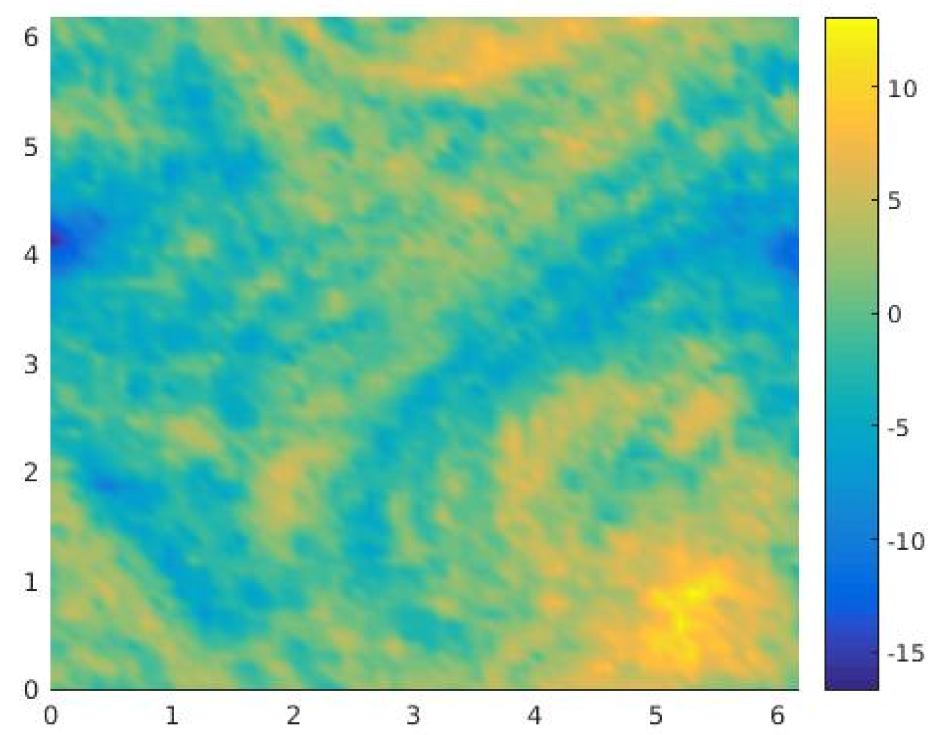
- After appropriate projections, truncations and time discretisation as in [2], we get the following Euler scheme to simulate from:

$$V^{L,M}_{m+1,k} = e^{-\nu\lambda_k h} V^{L,M}_{m,k} + \nu^{-1}\lambda_k^{-1}(1 - e^{-\nu\lambda_k h}) \left\langle \psi_k, F_L(V^{L,M}_m) \right\rangle + \left( \frac{\sigma_k^2}{2\nu\lambda_k}(1 - e^{-2\nu\lambda_k h}) \right)^{1/2} \chi_{k,m}$$

where $\chi_{k,m} = N(0,1) + iN(0,1)$ is a standard (complex) normal random variable.



Vector field at $t = 5$



Vorticity plot at $t = 5$

## Particle filtering in High dimensional Filtering problems

### Background

- Particle filtering is a sequential Monte Carlo (SMC) method that approximates $\mathcal{L}aw(V_{0:t}|Y_{0:t})$ with a particle approximation like $\frac{1}{n}\sum_{i=1}^{n} \delta_{V^i_{0:t}}$.
- Typical two step procedure
  - Weight particle using the observation likelihood, $g(Y_t|V_t^i)$.
  - Resampling or Selection step: once each particle has been assigned a normalised weight, resample from the particles to get equally weighted samples.
  - Use adaptive MCMC steps to jitter the particle population and reinsert lost diversity.
- Standard particle filters require very high number of particles to get good results (exponential with dimension size).
- When SPDEs are used for the signal we need a particle filter robust to mesh refinement and high dimensional state spaces.

### Particle filtering for SPDEs

- We will use a more elaborate scheme
  - Weight particle using adaptative tempering: add intermediate steps within each observations defined as:

$$g_{n,m}(V_n, Y_n) = p(Y_n|V_n)^{\phi_{n,m}}$$

where $0 < \phi_{n,1} < \ldots < \phi_{n,\tau_n} = 1$ chosen on the fly to optimize the number of steps.
  - Resample particle according to the normalised weights.
  - Use particle adapted MCMC kernels based on pre-conditioned Crank-Nickolson scheme of [4].
- Scheme is robust to mesh refinement even in infinite dimensions!
- Can improve the mixing of the above MCMC scheme by using proposals with information from particles
- This will be an extension of the ideas in [3], where a related SMC method was used for an inverse problem.

**Supervisors:** N.Kantas, D. Crisan, H. Brindley. **Collaborators:** A. Beskos, A. Jasra.

## References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[3] Nikolas Kantas, Alexandros Beskos, and Ajay Jasra. Sequential monte carlo methods for high-dimensional inverse problems: A case study for the navier–stokes equations. *SIAM/ASA Journal on Uncertainty Quantification*, 2(1):464–489, 2014.

[4] Andrew M Stuart. Inverse problems: a bayesian perspective. *Acta Numerica*, 19:451–559, 2010.