What Can Entropy in the Earth System Tell Us?

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Thermodynamic Underpinnings

Entropy is a measure of the number (Ω) of microstates (n) available to a system in a certain observable macrostate.

$$S = k_B \ln \Omega \quad \text{or} \quad -k_B \sum_n p(n) \log p(n)$$

It increases perpetually, as heat flows into cold milk from tea or salt diffuses in the ocean, providing a direction to all the developments we observe in our natural world.

Equilibrium is reached between two objects when no allowed exchange (of heat, E, say) could increase the total entropy (S). At this state, temperatures (T) are the same:

$$\frac{1}{T} := \frac{\partial S}{\partial E}$$

Heat is not the only transport between objects which is entropy maximising: molecular concentration of salt or water vapour will also equalise when allowed. However, radiation carrying thermal energy is the main quantity which enters and leaves the planet.

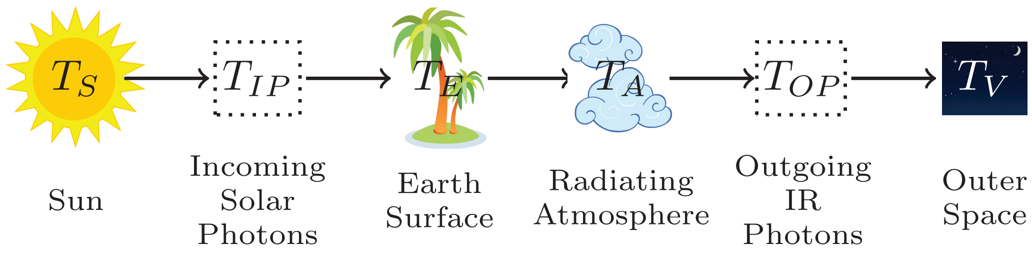
Radiation carries energy and entropy. For a black body radiating into a vacuum, energy flux (F) and entropy flux (J_s) are given by:

$$F = \sigma T^4 \quad \text{and} \quad J_s = \frac{4}{3}\sigma T^3$$

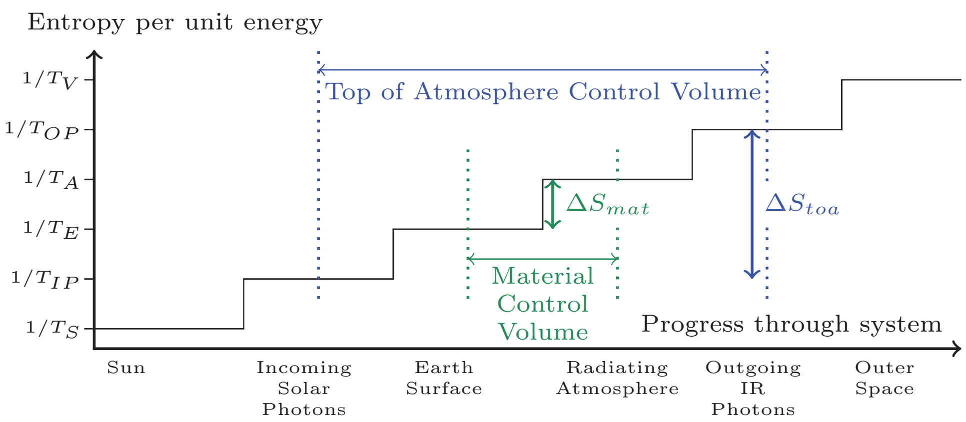
while for more complicated spectra, entropy must be calculated as a function of wavelength and effective temperature.

The System

Heat transfer is a two-way arrangement. What are the pairings in our system?



At every step the temperature decreases, so we can track a packet of energy as it gains entropy.



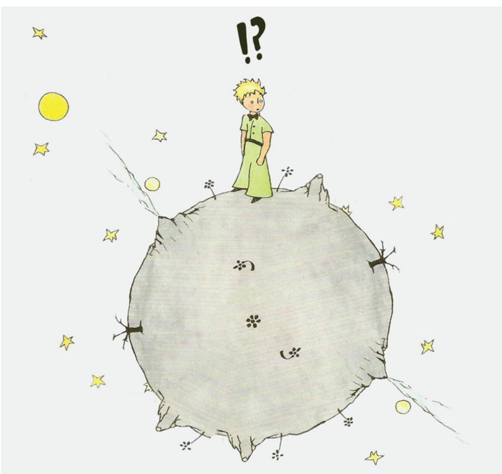
With the satellite data and GCM runs that we have, what can we calculate about the portion on Earth?

Aliens (or our satellites) can't know it all.

Question: Imagine two worlds of the same size circling the same sun at the same distance. Can aliens tell the difference between the following?

1) a rock absorbing and emitting as a black body.

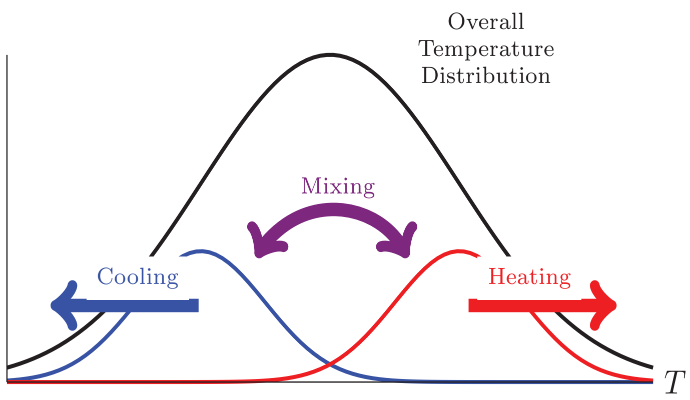
2) a planet similar to our earth with an absorbing surface topped by an atmosphere with an extreme greenhouse effect, making it transparent to radiation at solar wavelengths but a black body otherwise.



Answer: Not without invading. To an outside observer, these planets look the same in terms of emission temperature and entropy fluxes. However, only one has a temperature difference that can sustain ordered circulation patterns and life under its apparently black exterior.

Temperature carries the key.

Question: How much entropy is produced by all the irreversible processes between where sunlight is absorbed and thermal radiation leaves for space? (The material control volume.)



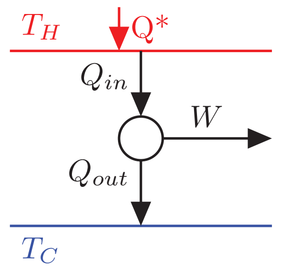
Answer: If we approximate the system as being unchanging, with the temperature, velocity and even water content fields steady in time thanks to an exact balance of forcing and dissipation, it is physically indistinguishable from a system in a universe where all non-conservative processes are turned off so the fields automatically sustain themselves, but heat is transferred directly (magically) from input (red) to output (blue) locations. Entropy production can now be calculated in this simplified case as it involves only one step:

$$\Delta S = Q\left(\frac{1}{T_{out}} - \frac{1}{T_{in}}\right)$$

Climates and efficiency.

Question: What is the climate efficiency? Per unit energy provided by the sun, Q*, how much work can be done in our climate system?

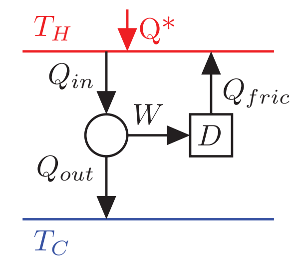
Answer: In a Carnot Engine, the amount of work that can be produced and exported due to exchanging heat from a hot to a cold reservoir is a function of the temperature difference as shown below.



$$\frac{Q_{in}}{T_H} = S_{in} \le S_{out} = \frac{Q_{out}}{T_C}$$

$$W = Q_{in} - Q_{out} \le Q^*\left(1 - \frac{T_C}{T_H}\right)$$

Since in the climate all the work generated is ultimately dissipated back into heat in order to keep total KE steady, it only exists temporarily. From a thermodynamic point of view, the same unit of energy could be converted to work and dissipated in the hot reservoir repeatedly, for example, increasing the apparent efficiency for the same temperature difference, until a net Q* has been transferred to the cold.



$$Q^* = Q_{in} - Q_{fric} = Q_{out}$$

$$W = Q_{in} - Q_{out} \le Q^*\left(\frac{T_H}{T_C} - 1\right)$$

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