Eddy-Induced Oceanic Material Transport and Mixing: The Model

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Poor assumptions made by current general circulation models (GCMs)

Eddy scales are frequently smaller than the grid resolution used by GCMs as its too computationally costly to further refine the grid scale. Therefore **eddy effects must be parameterised**. Currently eddy-induced transport is parameterised according to **downgradient diffusion** as suggested by Taylor (1921).
As a result transport is assumed to be

- homoegenous,
- isotropic,
- and diffusive.

Numerical experiments and physical obseervations have shown that this in fact is not the case.

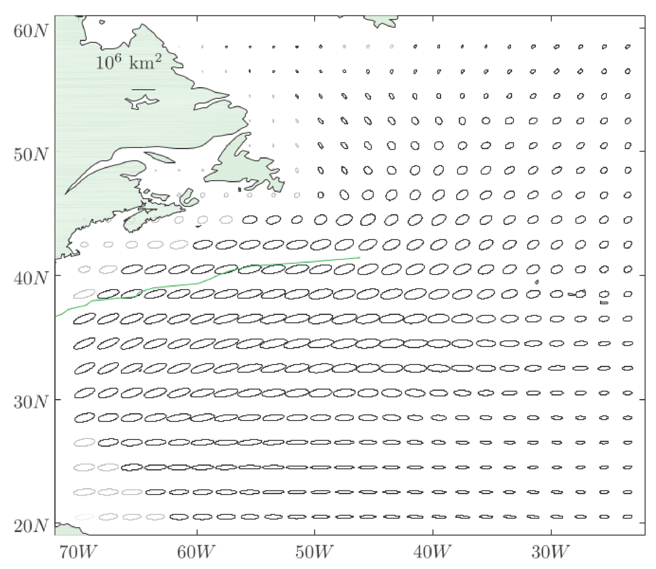

Figure: Spreading ellipses in the North Atlantic due to eddy-only velocity field approximated using satellite altimetry. Demonstrates anisotropy (Rypina et al.).

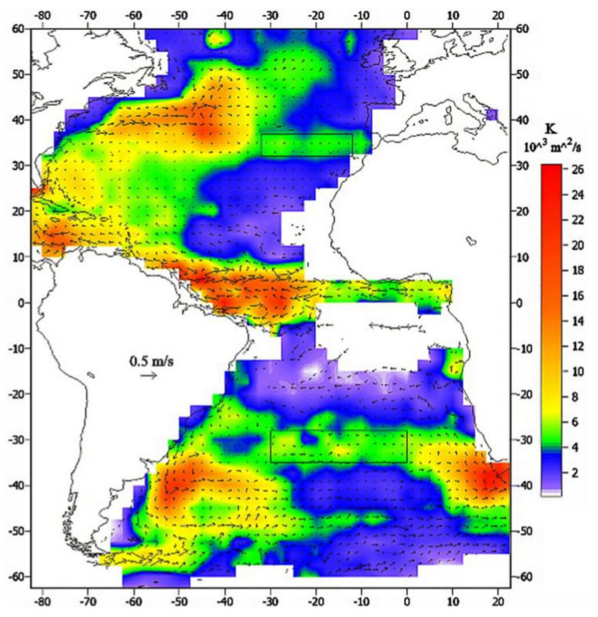

Figure: Lateral diffusivities as approximated using surface drifers. The arrows indicate velocity vectors. Demonstrates inhomogeniety (LaCasce).

The Dynamical Model

The transport model will be applied to a **2 layer quasi geostrophic model** which simulates a simple but realistic turbulent meandering jet.
The dynamical model relies on **potential voriticity anomaly inversion**:

$$q_1 = \nabla^2\psi_1 - S_1(\psi_1 - \psi_2), \quad q_2 = \nabla^2\psi_2 - S_2(\psi_2 - \psi_1), \tag{1}$$

where S_1 and S_2 are stratification parameters.
Bottom friction is varied in order to produce two different parameter regimes simulating a well defined jet and a more energetic, weakly defined wildly meandering jet.

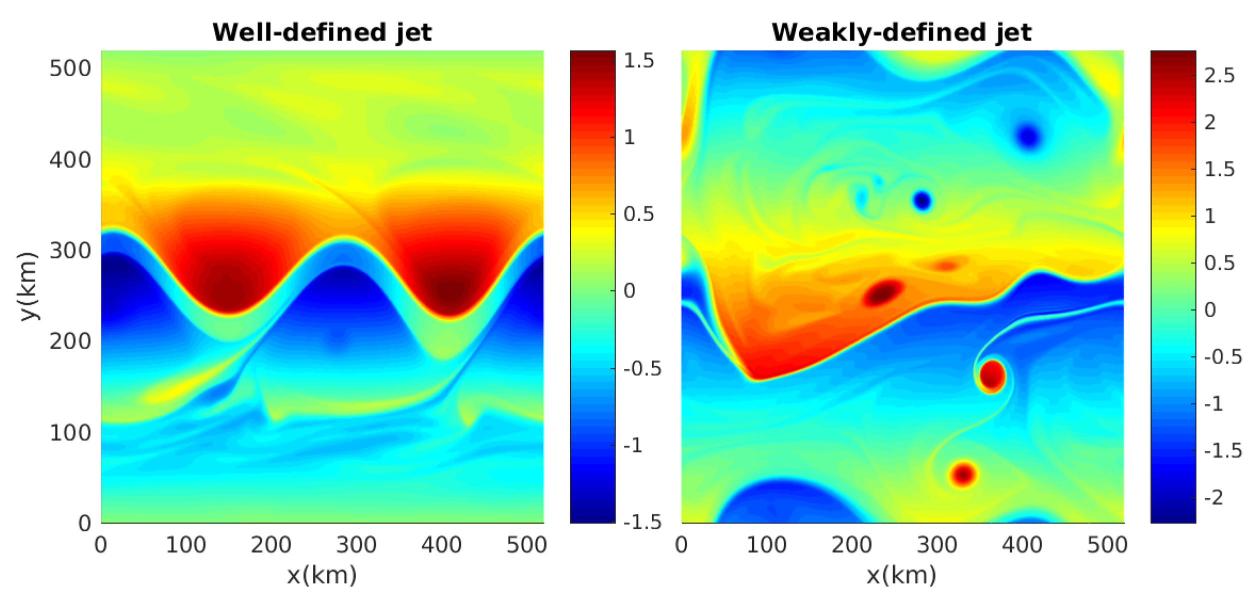

Figure: Potential vorticity anomaly in the top layer for the two parameter regimes

Lagrangian Statistics

Two types of Lagrangian statistics will be considered: those that measure **transport** and those that measure **mixing**. Transport only concerns the movement of individual particles, whereas mixing is about the movement of particles relative to eachother and can be used to diagnose barriers.
The most simple single particle statistic is that of **single-particle dispersion**:

$$D_x(t) = \frac{1}{N}\sum_{n=1}^{N}[x_n(t) - X(t)]^2, \quad D_y(t) = \frac{1}{N}\sum_{n=1}^{N}[y_n(t) - Y(t)]^2, \tag{2}$$

where $X(t)$ and $Y(t)$ are the ensemble mean zonal and meridional displacements at time t respectively, N is the number of particles and $x_n(t)$ and $y_n(t)$ are the zonal and meridional displacements of the nth particle at time t.
This statistic is very useful as it can be used to descirbe anisotropic and non-diffusive behaviours. **Fitting the single-particle dispersion to time using a power law**, $D \sim t^{\alpha}$, can diagnose its spreading rate:

- If $0.8 < \alpha < 1.2$, transport is said to be roughly **diffusive**,
- if $\alpha < 0.8$, transport is said to be **subdiffusive**,
- if $\alpha > 1.2$, transport is said to be **superdiffusive**.

The ratio of D_x to D_y can be used to diagnose the level of anistropy.

Chaos is defined as a flows sensitivity to initial conditions. This can be quantified by way of **Lyapunov exponents**.
If $d\mathbf{X}_t$ is the separation between two particles at time t and $d\mathbf{X}_o$ is their initial separation then,

$$|d\mathbf{X}_t| \approx e^{\lambda t}|d\mathbf{X}_0|, \tag{3}$$

where λ is the Lyapunov exponent. λ hence measures how fast particles separate from each other. **A positive Lyapunov exponent indicates Lagrangian chaos**.

The Transport Model

A particle's trajectory can be described as follows,

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{u}(t), \tag{4}$$

where $\mathbf{x}(t)$ is the particle's location at time t and $\mathbf{u}(t)$ is its velocity. This equation describes a continues velocity field, whereas we only know the velocity field at discrete locations. Therefore the **velocity field must by interpolated spatially** in order to approximate the velocity at location $\mathbf{x}$ while preserving the **non-divergence property**. This is done by constructing a polynomial which approximates the streamfunction, which is then differentiated analytically to find the velocity, since

$$-\frac{\partial\psi}{\partial y} = u, \quad \frac{\partial\psi}{\partial x} = v. \tag{5}$$

Time integration is then performed using the **Runge-Kutta fourth order method**.

Some results

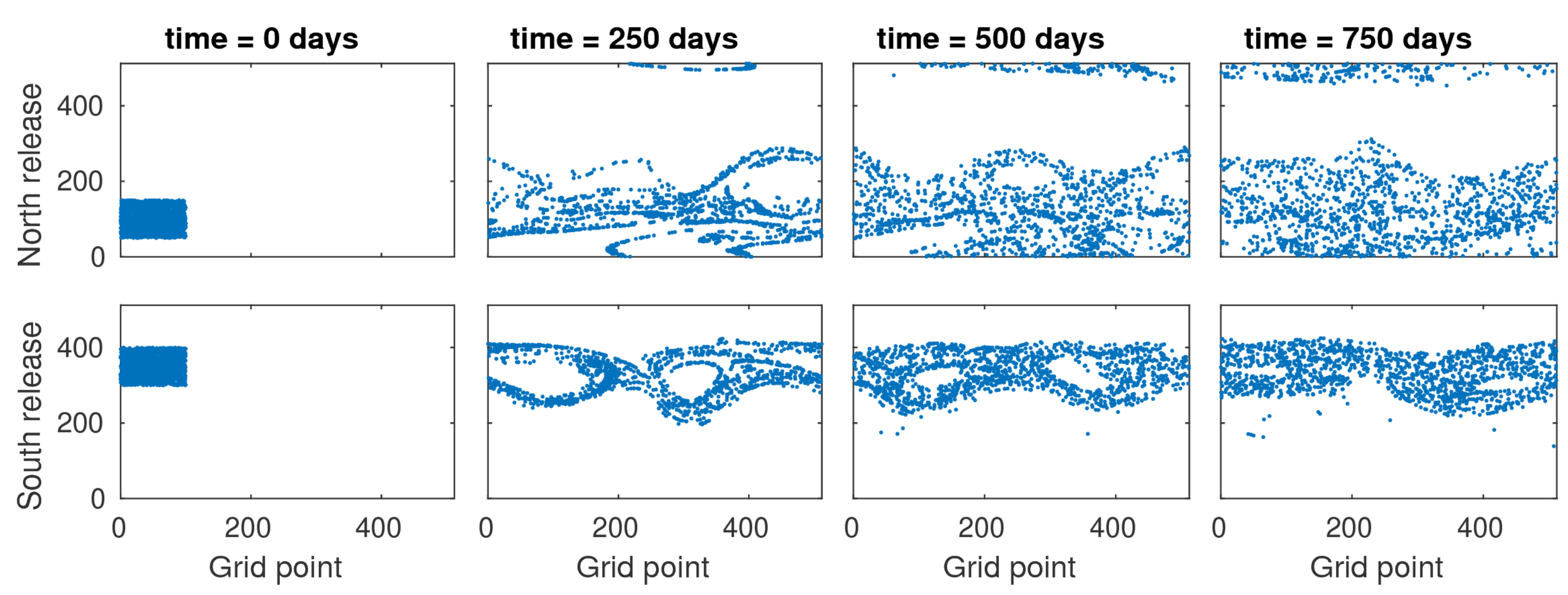

Figure: Scatter plot of particles released from different locations at different times as advected by the full velocity field of first parameter regime in the top layer

The scatter plots show the evolution of particles from different release locations: north and south of the jet. As expected, **the jet acts as a partial barrier to transport** with very few particles crossing the jet core.
Furthermore **eddying structures** in the vicinity of the jet core can also be diagnosed by regions where few particles have explored.
A more rigorous analysis would be carried out by measure the statistics as described.

References

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